

Effective attraction induced by repulsive interaction in a spin-transfer system

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In magnetic systems with dominating easy-plane anisotropy the magnetization can be described by an effective one dimensional equation for the in-plane angle. Re-deriving this equation in the presence of spin-transfer torques, we obtain a description that allows for a more intuitive understanding of spintronic devices' operation and can serve as a tool for finding new dynamic regimes. A surprising prediction is obtained for a planar "spin-flip transistor": an unstable equilibrium point can be stabilized by a current induced torque that further repels the system from that point. Stabilization by repulsion happens due to the presence of dissipative environment and requires a Gilbert damping constant that is large enough to ensure overdamped dynamics at zero current.

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In physics, there are cases where due to the presence of complex environment a repulsive force can lead to actual attraction of the entities. A well known example is a superconductor, where the Cooper pairs are formed from electrons repelled by the Coulomb forces due to the dynamical elastic environment. Here we report a phenomena of effective attraction induced by the repulsive spin-transfer torque in the presence of highly dissipative environment. The spin-transfer effect producing the repulsive torque is a non-equilibrium interaction that arises when a current of electrons flows through a non-collinear magnetic texture [1, 2, 3]. This interaction can become significant in nanoscopic magnets and is nowadays studied experimentally in a variety of systems. Its manifestations - either current induced magnetic switching [4] or magnetic domain wall motion [5] - serve as an underlying mechanism for a number of suggested memory and logic applications.

Here we consider a conventional spin-transfer device consisting of a magnetic polarizer (fixed layer) and a small magnet (free layer) with electric current flowing from one to another (Fig 1). Both layers can be described by a macro-spin model due to large exchange stiffness. The free layer is influenced by the spin transfer torque, while the polarizer is too large to feel it. Magnetic dynamics of the free layer is described by the Landau-Lifshitz-Gilbert (LLG) equation with the spin transfer torque term [2, 6].

The solutions of LLG are easy to find for the simplest easy axis magnetic anisotropy of the free layer. There exists a critical current at which the free layer either switches between the two minima of magnetic energy, or goes into a state of permanent precession, powered by the current source [2, 6, 7]. The same basic processes happen in the case of realistic anisotropies, however the complexity of the calculations increases substantially. In a nanopillar device [8] one additionally finds that stabilization of magnetic energy maxima is possible ("canted

states" [6]) and that multiple precession modes exist with transitions between them happening as the current is increased [7, 9, 10]. The anisotropy of a nanopillar device is a combination of a magnetic easy plane and magnetic easy axis directed in that plane. Experimentally, the easy plane anisotropy energy is usually much larger than the easy axis energy, i.e. the system is in the regime of a planar spintronic device [11] (Fig. 1). This limit of dominating easy plane energy is characterized by another simplification of the dynamic equations [12, 13], which comes not from the high symmetry of the problem, but from the existence of a small parameter: the ratio of the energy modulation within the plane to the easy plane energy. The deviation of the magnetization from the plane becomes small, making the motion effectively one dimensional.

In this paper we present a general form of effective planar equation describing a macrospin free layer in the presence of spin transfer torques. Its relationship to the first order expansion in the current magnitude used in Ref. 13 is discussed at the end. We then use this equation to study the "spin-flip transistor": a planar device in which the spin polarizer is perpendicular to the direction favored by the magnetic anisotropy energy. It was predicted [14] that the competition between the anisotropy and spin transfer torques leads to a 90 degrees jump of

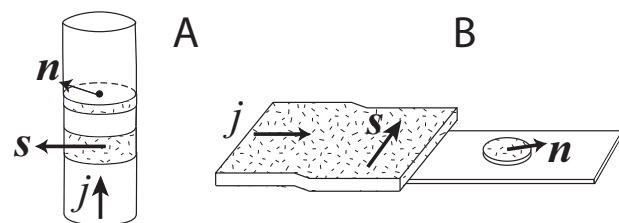


FIG. 1: Planar spin-transfer devices. Hashed parts of the devices are ferromagnetic, white parts are made from a non-magnetic metal.

the magnetization at the critical current. Whether the jump happens into the parallel or antiparallel state with respect to the polarizer is determined by the direction of the current.

Here it is shown that the behavior of the spin-flip transistor is more complicated than expected from the simple picture above. Namely, the current inducing a jump into the parallel direction can also stabilize the antiparallel direction. This conclusion is certainly counter-intuitive because the spin torque repels the magnetization from this already unstable saddle point of the energy. However, a combination of two destabilizing torques manages to result in a stable equilibrium. We will see that this happens due to the dissipation terms and a sufficiently large (but still small compared to unity) Gilbert damping constant is required to observe the phenomena.

The magnetization of the free layer $\mathbf{M} = M\mathbf{n}$ has a constant absolute value M and a direction given by a unit vector $\mathbf{n}(t)$. The LLG equation [2, 6] reads:

$$\dot{\mathbf{n}} = \frac{\gamma}{M} \left[-\frac{\delta E}{\delta \mathbf{n}} \times \mathbf{n} \right] + u(\mathbf{n})[\mathbf{n} \times [\mathbf{s} \times \mathbf{n}]] + \alpha[\mathbf{n} \times \dot{\mathbf{n}}] . \quad (1)$$

Here γ is the gyromagnetic ratio, $E(\mathbf{n})$ is the magnetic energy of the free layer, and α is the Gilbert damping constant. The second term on the right is the spin transfer torque, where \mathbf{s} is a unit vector along the direction of the polarizer, and the spin transfer strength $u(\mathbf{n})$ is proportional to the electric current I [6, 13]. In general, spin transfer strength is a function of the angle between the polarizer and the free layer $u(\mathbf{n}) = f[(\mathbf{n} \cdot \mathbf{s})] I$, with the function $f[(\mathbf{n} \cdot \mathbf{s})]$ being material and device specific. Equation (1) can be written in polar angles $(\theta(t), \phi(t))$:

$$\begin{aligned} \dot{\theta} + \alpha \dot{\phi} \sin \theta &= -\frac{\gamma}{M \sin \theta} \frac{\partial E}{\partial \phi} + u(\mathbf{s} \cdot \mathbf{e}_\theta) \equiv F_\theta , \\ \dot{\phi} \sin \theta - \alpha \dot{\theta} &= \frac{\gamma}{M} \frac{\partial E}{\partial \theta} + u(\mathbf{s} \cdot \mathbf{e}_\phi) \equiv F_\phi , \end{aligned} \quad (2)$$

with tangent vectors $\mathbf{e}_\phi = [\hat{z} \times \mathbf{n}]/\sin \theta$, $\mathbf{e}_\theta = [\mathbf{e}_\phi \times \mathbf{n}]$. The easy plane is chosen at $\theta = \pi/2$, and the magnetic energy has the form $E = (K_\perp/2) \cos^2 \theta + E_r(\theta, \phi)$, where E_r is the “residual” energy. In the planar limit, $K_\perp \rightarrow \infty$, the energy minima are very close to the easy plane and the low energy solutions of LLG have the property $\theta(t) = \pi/2 + \delta\theta$ with $\delta\theta \rightarrow 0$. Equations (2) can then be expanded in small parameters $|E_r|/K_\perp \ll 1$, $|u(\mathbf{n})|/K_\perp \ll 1$. Assuming time-independent u and \mathbf{s} we obtain an effective equation of the in-plane motion

$$\frac{1}{\omega_\perp} \ddot{\phi} + \alpha_{eff} \dot{\phi} = -\frac{\gamma}{M} \frac{\partial E_{eff}}{\partial \phi} , \quad (3)$$

which has the form of the Newton’s equation of motion for a particle in external potential $E_{eff}(\phi)$ with a variable viscous friction coefficient $\alpha_{eff}(\phi)$. The expressions for the effective friction and energy are

$$\begin{aligned} \alpha_{eff}(\phi) &= \alpha - (\Gamma_\phi + \Gamma_\theta)/\omega_\perp , \\ \Gamma_\phi &= (\partial F_\phi / \partial \phi)_{\theta=\pi/2} , \quad \Gamma_\theta = (\partial F_\theta / \partial \theta)_{\theta=\pi/2} , \end{aligned} \quad (4)$$

and

$$\begin{aligned} E_{eff}(\phi) &= E_r(\pi/2, \phi) + \Delta E(\phi) , \\ \Delta E &= -\frac{M}{\gamma} \int_{\pi/2}^\phi \left[u(\mathbf{n})(\mathbf{s} \cdot \mathbf{e}_\theta) - \frac{\Gamma_\theta}{\omega_\perp} F_\phi \right]_{\theta=\pi/2} d\phi' . \end{aligned} \quad (5)$$

Equation (3) with definitions (4,5) gives a general description of a planar device in the presence of spin transfer torque. At non-zero current the effective friction can become negative (see below), and the effective energy is not necessarily periodic in ϕ (e.g. in the case of “magnetic fan” [13, 15]). Physically this reflects the possibility of extracting energy from the current source, and thus developing a “negative dissipation” in the system.

In many planar devices the polarizer direction \mathbf{s} lies in the easy plane, $\theta_s = \pi/2$, with a direction defined by the azimuthal angle ϕ_s . At the same time the residual energy has a property $(\partial E_r / \partial \theta)_{\theta=\pi/2} = 0$, i.e. does not shift the energy minima away from the plane. We will also use the simplest form $f[(\mathbf{n} \cdot \mathbf{s})] = \text{const}$ for the spin transfer strength. A more realistic function will not change the result qualitatively and can be easily used if needed. With these restrictions the effective friction and the energy correction get the form:

$$\begin{aligned} \alpha_{eff} &= \alpha + \frac{2u \cos(\phi_s - \phi)}{\omega_\perp} \\ \Delta E &= -\frac{Mu^2}{2\gamma\omega_\perp} \cos^2(\phi_s - \phi) . \end{aligned} \quad (6)$$

In a spin-flip transistor the polarizer direction is given by $\phi_s = \pi/2$. Following Ref. 14, we consider in-plane anisotropy energy $E_r(\pi/2, \phi) = -(K_\parallel/2) \cos^2 \phi$ corresponding to an easy axis. Then the effective friction is $\alpha_{eff} = \alpha + (2u \sin \phi)/\omega_\perp$ and effective energy equals $(\gamma/M)E_{eff} = -[(\omega_\parallel - u^2/\omega_\perp)/2] \cos^2 \phi + \text{const}$ with $\omega_\parallel = \gamma K_\parallel/M$. Equilibrium points $\phi = 0, \pm\pi/2, \pi$ are the minima and maxima of the effective energy, and do not depend on u . Stability of any equilibrium in one dimension depends on whether it is a minimum or a maximum of E_{eff} and on the sign of α_{eff} at the equilibrium point. It is easy to check, that out of four possibilities only an energy minimum with $\alpha_{eff} > 0$ is stable. In the case of a spin-flip transistor the energy landscape changes above a threshold $|u| > \sqrt{\omega_\parallel \omega_\perp}$: the energy minima at $\phi = 0, \pi$ become maxima, and, vice versa, the energy maxima at $\phi = \pm\pi/2$ switch to minima. Effective friction at $\phi = 0, \pi$ is positive independent of u , while at $\phi = \pm\pi/2$ it changes sign at $u = \mp\alpha\omega_\perp/2$.

The behavior of the spin-flip transistor is summarized in a switching diagram Fig. 2 plotted on the plane of the material characteristic α and the experimental parameter $u \sim I$. For definiteness we will discuss a current with $u > 0$. The effect of the opposite current is completely symmetric. For small values of Gilbert damping one observes stabilization of the $\phi = \pi/2$ (parallel) equilibrium

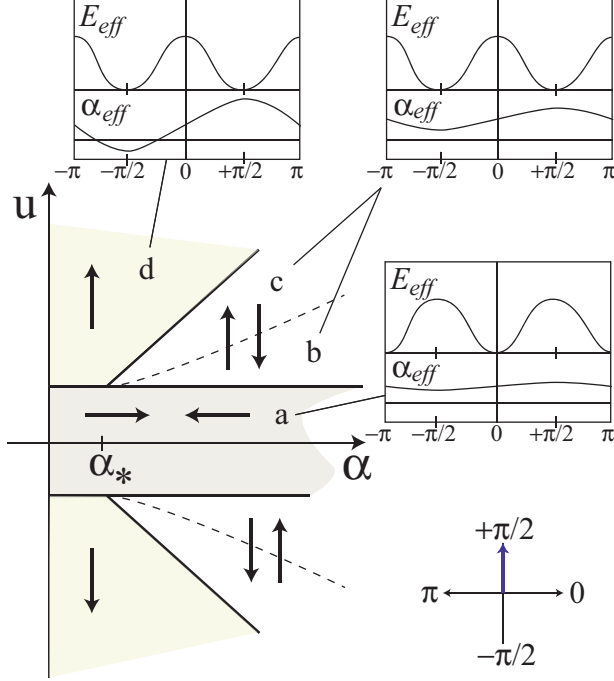


FIG. 2: Switching diagram of the spin-flip transistor. In each zone one or two arrows show the possible stable directions of the free layer magnetization. Directions of the easy axis and spin polarizer are defined in the right bottom corner. Angular dependencies of α_{eff} and E_{eff} are given in insets. Stable sub-regions “b” and “c” differ in overdamped vs. underdamped approach to the equilibrium.

to which the spin torque attracts the magnetization of the free layer, while the opposite (antiparallel) direction remains unstable. This is in accord with the results of Ref. 14. However, when the damping constant is larger than the critical value $\alpha_* = 2\sqrt{\omega_{||}/\omega_{\perp}}$, a window of stability of the antiparallel equilibrium opens on the diagram. Since $\alpha \ll 1$, a sufficiently large easy plane energy is required to achieve $\alpha_* < \alpha \ll 1$.

If one thinks about the stability of the $(\theta, \phi) = (\pi/2, -\pi/2)$ equilibrium for $u > 0$ in terms of Eq. (1), this prediction seems completely unexpected. The anisotropy torques do not stabilize this equilibrium because it is a saddle point of the total magnetic energy E , and the added spin transfer torque repels \mathbf{n} from this point as well. The whole phenomena may be called “stabilization by repulsion”. To check the accuracy of the planar approximation (3), the result was verified using the LLG equations (2) with no approximations for the axis-and-plane energy $E = (K_{\perp}/2)\cos^2\theta - (K_{||}/2)\sin^2\theta\cos^2\phi$. Calculating the eigenvalues of the linearized dynamic matrices [6] at the equilibrium points $(\pi/2, \pm\pi/2)$ we obtained the same switching diagram and confirmed the stabilization of the antiparallel direction. Typical trajectories $\mathbf{n}(t)$ numerically calculated from the LLG equation with no approximations are shown in Fig. 3 to illus-

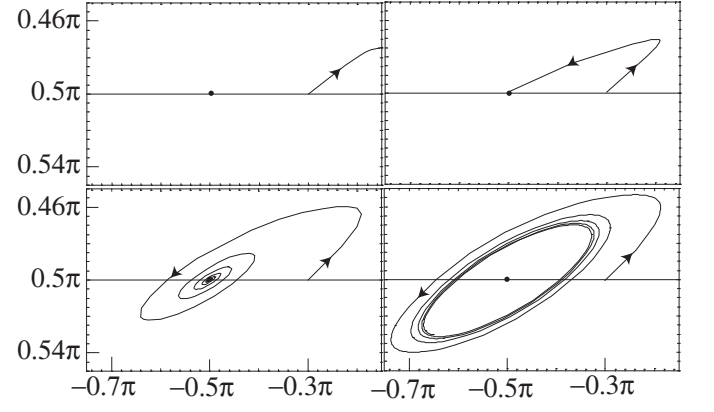


FIG. 3: Typical trajectories of $\mathbf{n}(t)$ for $\omega_{||}/\omega_{\perp} = 0.01$, $\alpha = 1.5\alpha_*$. The plot labels correspond to the regions in Fig. 2, the current magnitude is given in the units of $u/\sqrt{\omega_{||}/\omega_{\perp}}$ and we look at the stability of the $\phi = -\pi/2$ equilibrium: (a) 0.93, unstable (b): 1.08, stabilized with overdamped approach (c): 1.38, stable, but with oscillatory approach (d): 1.53, unstable; a stable cycle is formed around the equilibrium.

trate the predictions. At $u > \sqrt{\omega_{||}\omega_{\perp}}$ the $\phi = -\pi/2$ equilibrium is stabilized. In accord with the predictions of Eqs. (3),(6), the wedge of its stability consists of two regions (b) and (c) characterized by overdamped and underdamped dynamics during the approach to the equilibrium. The dividing dashed line is given by $u = \omega_{||}/\alpha + \alpha\omega_{\perp}/4$. It was checked that small deviations of the polarizer \mathbf{s} from the $(\pi/2, \pi/2)$ direction do not change the behavior qualitatively. Larger deviations eventually destroy the effect, especially the out-of-plane deviation which produces the “magnetic fan” effect [15] leading to the full-circle rotation of ϕ in the plane.

As the current is further increased to $u > \alpha\omega_{\perp}/2$, the antiparallel state loses stability and the trajectory approaches a stable precession cycle (Fig. 3(d)). The existence of the precession state is easy to understand from (3) viewed as an equation for a particle in external potential. Just above the stability boundary the effective friction $\alpha_{eff}(\phi)$ is negative in a small vicinity of $\phi = -\pi/2$, and positive elsewhere. Within the $\alpha_{eff} < 0$ region the dissipation is negative and any small deviation from the equilibrium initiates growing oscillations. As their amplitude exceeds the size of that region, part of the cycle starts to happen with positive dissipation. Eventually the amplitude reaches a value at which the energy gain during the motion in the $\alpha_{eff} < 0$ region is exactly compensated by the energy loss in the $\alpha_{eff} > 0$ region: thus a cycle solution emerges. The effective planar description allows for the analysis of the further evolution of the cycle with transitions into different precession modes, which will be a subject of another publication.

The fact that $\alpha > \alpha_*$ condition is required for the stabilization means that dissipation terms play a crucial role entangling two types of repulsion to produce a net attraction to the reversed direction. Note that an interplay of a strong easy plane anisotropy and dissipation terms produces unexpected effects already in conventional ($u = 0$) magnetic systems. The effective planar equation (3) at $u = 0$ was discussed in Ref. 12. It was found that the same threshold α_* represents a boundary between the oscillatory and overdamped approaches the equilibrium. Above α_* the familiar precession of a magnetic moment in the anisotropy field is replaced by the dissipative motion directed towards the energy minimum. When the easy plane anisotropy is strong enough to ensure $\alpha \gg \alpha_*$, one can drop the second order time derivative term in Eq. (3) and use the resulting first order dissipative equation. In the presence of spin transfer, $\alpha_{eff}(\phi, u)$ depends on the current and can assume small values even for $\alpha \gg \alpha_*$, thus no general statement about the $\ddot{\phi}$ term can be made.

The simplest easy axis energy expression $E_r(\pi/2, \phi) = -(K_{||}/2)\cos^2\phi$ happens to have the same angular dependence as $\Delta E(\phi)$ given by Eq. (6). Due to this special property the energy profile flips upside down at $u = \sqrt{\omega_{||}\omega_{\perp}}$. For a generic $E_r(\pi/2, \phi)$ with minima at $\phi = 0, \pi$ and maxima at $\phi = \pm\pi/2$ the nature of equilibria will change at different current thresholds. This will make the switching diagram more complicated, but will not affect the stabilization by repulsion phenomena. Similar complications will be introduced by a generic $f[(\mathbf{n}\cdot\mathbf{s})]$ angular dependence of the spin transfer strength.

In Ref. 13 the known switching diagram for the collinear ($\phi_s = 0$) devices [6, 9, 10] were reproduced by equation (3) with $E_{eff} = E_r(\pi/2, \phi)$. The ΔE term (6) was dropped as being second order in small u . This approximation gives a correct result for the following reason. In a collinear device $(\gamma/M)E_{eff} = -[(\omega_{||} + u^2/\omega_{\perp})/2]\cos^2\phi + \text{const}$ and the current never changes the nature of the equilibrium from a maximum to a minimum. Consequently, dropping ΔE does not affect the results. As was already noted in Ref. 13, the first order expansion in u is insufficient for the description of a spin-flip transistor, where the full form (6) is required.

In summary, we derived a general form of the effective planar equation (3) for a macrospin free layer in the presence of spin transfer torque produced by a fixed spin-polarizer and time-independent current. Qualitative understanding of the solutions of planar equation is obtained by employing the analogy with a one-dimensional mechanical motion of a particle with variable friction coefficient in an external potential. The resulting predictive power is illustrated by the discovery of the stabilization by repulsion phenomena in the spin-flip device. Such stabilization relies on the form of the dissipative torques in the LLG equation and happens only for a large enough Gilbert damping constant. The new stable state

and the corresponding precession cycle can be used to engineer novel memory or logic devices, and microwave nano-generators with tunable frequency.

To observe the phenomena experimentally, one has to fabricate a device with $\alpha > \alpha_*$, and initially set it into a parallel or antiparallel state by external magnetic field. Then the current is turned on and the field is switched off. Both states should be stabilized by a moderate current $\sqrt{\omega_{||}\omega_{\perp}} < u < \alpha\omega_{\perp}/2$, but cannot yet be distinguished by their magnetoresistive signals. The difference can be observed as the current is increased above the $\alpha\omega_{\perp}/2$ threshold: the parallel state will remain a stable equilibrium, while the antiparallel state will transform into a precession cycle and an oscillating component of magnetoresistance will appear.

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